Exercises: Transform Coding

The following tasks can be implemented in MATLAB.

1. Image compression uses the fact that image data contains a high degree of redundancy. Most images are not random collections of pixel values but underlie some form of structure. \[1\]

(a) Download the sample picture from the homepage [http://r0k.us/graphics/kodak/kodim23.html](http://r0k.us/graphics/kodak/kodim23.html)

Read the image with entries denoted here by 
\[ f(k, l) \quad \text{with } k = 1, 2, \ldots, \dim_1, \; l = 1, 2, \ldots, \dim_2, \]

where \( \dim_1 \) and \( \dim_2 \) are the image dimensions. Transform it to YCbCr space and use only the luma components for this exercise.

(b) Divide the image into nonoverlapping blocks of size \( 1 \times 2 \), and denote 
\[ (s_l, s_{l+1}) = (f(k, l), f(k, l + 1)) \quad k = 1, \ldots, \dim_1 \in \mathbb{R}^{\dim_1 \times 1} \]

for \( l = 1, \ldots, \dim_2 - 1 \) and 
\[ x = [s_1^T, s_3^T, \ldots, s_{\dim_2 - 1}^T] \in \mathbb{R}^{1 \times \dim_1 \cdot \dim_2}, \]

\[ y = [s_2^T, s_4^T, \ldots, s_{\dim_2}^T] \in \mathbb{R}^{1 \times \dim_1 \cdot \dim_2}, \]

assuming \( \dim_2 \) is an even number. Make a scatter plot of the adjacent pixel pairs \( (x, y) \in \mathbb{R}^{2 \times \dim_1 \cdot \dim_2} \).

(c) Transform \( (x, y) \) column-wise using the DCT matrix 
\[ D = (d_{i,j})_{i,j=0,1} \]

defined by 
\[ d_{i,j} = \alpha_i \cos\left(\frac{\pi(2j + 1)i}{2N}\right) \]

with \( \alpha_i = \frac{1}{\sqrt{N}} \begin{cases} 1 & \text{for } i = 0, \\ \sqrt{2} & \text{for } i > 0 \end{cases} \)

and \( N = 2 \) \[\text{(2, p.203)}\]. Then, make a scatter plot of the transformed pair \( \left(\frac{u}{v}\right) = D \left(\frac{x}{y}\right) \).

2. Assume an iid source with a Gaussian distribution, a high rate and uniform quantization for the random vector \( S = (S_1, S_2) \). Then, the transform coding gain (\[1\] p.29) is defined as 
\[ G_T = \frac{1}{2} \left( \frac{\sigma_{S_1}^2 + \sigma_{S_2}^2}{\sigma_{S_1}^2 \cdot \sigma_{S_2}^2} \right). \] \[ (1) \]

Now, assuming these requirements are fulfilled for our random vectors \( (X, Y)^T \) and \( (U, V)^T \) of which the pairs \( (x_i, y_i)^T, i = 1, \ldots, \dim_1 \cdot \frac{\dim_2}{2} \) respectively \( (u_i, v_i)^T, i = 1, \ldots, \dim_1 \cdot \frac{\dim_2}{2} \) represent realizations.
(a) Calculate the sum of the variances for \( (x, y) \) and for \( (u, v) \).

(b) Calculate the transform coding gain for \( (x, y) \) and for \( (u, v) \).

References
