Exercises: Elias Coding

The following tasks can be implemented in MATLAB.

1. Consider following iid source.

<table>
<thead>
<tr>
<th>symbol $a_k$</th>
<th>$p(a_k)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$a_0 = 'A'$</td>
<td>0.25</td>
</tr>
<tr>
<td>$a_1 = 'B'$</td>
<td>0.25</td>
</tr>
<tr>
<td>$a_2 = 'C'$</td>
<td>0.5</td>
</tr>
</tbody>
</table>

Let $\{i_1, i_2, i_3, i_4\}$ denote the enumeration of the interval sections. Then, the probabilities of the symbols $a_k$, $k \in \{0, 1, 2\}$ can be represented as intervals inside the unit interval as follows.

$$
\begin{align*}
A & \quad i_1 = 0 \\
B & \quad i_2 = 0.25 \\
C & \quad i_3 = 0.5 \\
D & \quad i_4 = 1
\end{align*}
$$

(a) Implement Elias Coding to generate a bitstream for a symbol sequence $s$. Update the interval boundaries in the following way:

- in case the symbol is $A$:
  $$i_4 \leftarrow i_4 - (p(B) + p(C)) \cdot (i_4 - i_1)$$
- in case the symbol is $B$:
  $$i_4 \leftarrow i_4 - p(C) \cdot (i_4 - i_1)$$
  $$i_1 \leftarrow i_2$$
- in case the symbol is $C$:
  $$i_1 \leftarrow i_3$$

Then, update the other interval sections correspondingly. At the end of the sequence (assuming its length is known), calculate the interval width $W$ of the current interval $[i_1, i_4]$. Compute the required bits $K$ to represent the interval by

$$K = \lceil -\log_2(W) \rceil.$$ 

Then, calculate the (decimal) representative $v$ of the interval $[i_1, i_4]$ via

$$v = [i_1 \cdot 2^K] \cdot 2^{-K}$$

and binarize $v$ to achieve a bitstream.

**Hint:** To approximate the binarization of a decimal fraction $d \in [0, 1)$ with accuracy of $K$ bits, consider following technique: Multiply $d$ by two. If $2d < 1$ note down 0. If $2d \geq 1$, note down 1 and update $d \leftarrow d - 1$. Repeat $K$ times.

(b) Encode the symbol sequence $s = 'CABAC'$. 

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**Elias Coding**